

RESEARCH ARTICLE

Data Science

Two level neuro-functional forecaster: A novel dynamic hybridization for functional data forecasting

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Abstract: With the advancement of technology, time series data are automatically collected without human intervention. As the data collection process becomes effortless, the next change encountered is to identify the best method to forecast time series data with high accuracies. In this regard, hybrid approaches have gained much attention where the strengths of two approaches can be combined to lessen the weaknesses of each individual approach. When exploring the features of time series data, some depict repetitive patterns and also data can be observed at several levels. The repeating curves can be considered as the higher level, whereas each individual observation can be considered as the lower level. Thus, in order to handle the data in a more effective way, the series can be handled at two levels by giving prominence to the features of each level separately. This paper proposes a novel algorithm named two level neuro-functional forecaster, which is capable of handling data at two levels, hybridizing a statistical approach with an artificial intelligence approach, to gain high accuracy levels. In addition, as this approach handles data at two levels, data sparsity at a particular level can be accommodated at the other level. To apply this algorithm to a real world dataset, electricity demand data in Sri Lanka was considered where the series consisted of daily load curves with repetitive pattern across the days. The proposed hybrid algorithm, outperforms the two approaches when individually used, with a MAPE of 3.324% for a year.

Keywords: Dynamic, forecasting, functional data analysis, hybrid, neural network.

INTRODUCTION

In most real-world situations, data are periodically collected and analyzed using various forecasting methods, and the structural aspect of such periodic data can be analyzed in different ways. One way of considering the structural aspect when forecasting, is through a sequence of curves that evolve over time. Alternatively, each point on the curve can be taken as a separate observation of the series forecast. As such, the forecasting can be done at a higher level in terms of curves or at a lower level in terms of observations.

A real-world example to illustrate the above-mentioned features of a time series can be commonly witnessed in the energy industry. A plot displaying the electricity demand within a day is usually named as a “daily load curve,” where for a specific time period, the electricity demand series shows a clear periodic pattern, possibly with a few exceptions based on other external factors. When considering load data, it can be commonly experienced that such data consist of several levels such as hourly, daily, weekly, or seasonally. Thus, the load forecasting models can be considered as an ideal application to examine how the structural aspect of a series can be considered in many ways when implementing

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forecasting models for improved accuracies. Currently, accurate and efficient short term load forecasting (STLF) models are gaining much attention, as even a minor improvement to such models can have a remarkable impact on the power and energy industry, and thereby the economy of a country.

In the recent past, much published work can be found utilizing hybrid models to forecast the load in a short term manner, showing promising results. When exploring such hybrid models in load forecasting, various approaches have been used. Some approaches are through combining two statistical techniques at two levels (Cho *et al.*, 2013) and some other approaches use two artificial intelligence approaches at a single level (Kavousi *et al.*, 2014; Bashir *et al.*, 2022). Studies also reveal that there are approaches combining a statistical technique with an artificial intelligence approach but based on data at a single level (Zhang, 2003; Nie *et al.*, 2012).

The model proposed by Bashir *et al.* have utilized a Prophet model and a long short-term memory (LSTM) model to predict load, in Belgium. They have used data at a single level where the hybridization is to use a LSTM model for the residuals resulting from the Prophet model. The results indicated that their hybrid models outperform the LSTM, Prophet model and ARIMA models when used on their own (Bashir *et al.*, 2022). Fard and Akbari-Zadeh (2014) have proposed a hybrid forecasting method based on the wavelet transform, autoregressive integrated moving average (ARIMA), and artificial neural network (ANN) for short-term load forecasting of the Fars Electric Power Company in Iran. The three models used in this study had been combined in such a way that the linear nature of the series has been captured using the ARIMA model, whereas the nonlinear patterns have been captured by the ANN models, and the wavelet for transformation purposes to avoid overfitting problems. They have shown that when the methods are combined, the accuracy level increases substantially (Fard & Akbari-Zadeh, 2014). By utilizing two artificial intelligence approaches along with regional level temperature data, Barman, Choudhury and Sutradhar have shown better accuracies when forecasting the load. They have used support vector machine (SVM) and the grasshopper optimization algorithm (GOA) to implement the hybrid model (Barman *et al.*, 2018).

As opposed to the above work where data are at a single level, Cho *et al.* (2013) have forecasted the electricity demand of a French energy company using a hybrid approach with data at two levels. Their approach combined a general additive model to capture the overall trend and seasonality, and a curve linear regression model

to capture the dependence structure, across consecutive daily loads. It was revealed that four versions of the hybrid models have been tested and the hybrid models have resulted in very low errors compared to using individual approaches.

The use of functional data analysis techniques has grown rapidly in relation to statistical analysis for clustering (Jacques & Preda, 2014), prediction (Antoch *et al.*, 2010; Shan, 2013; Jornaz, 2016), and system investigation (Gubian *et al.*, 2014), proving to generate promising results. However, there are only a few functional modelling approaches that have been applied to electricity demand on short term, medium term (Antoch *et al.*, 2010) and very short term (Shan, 2013) for forecasting. Antoch *et al.* (2010) have also proved, through their research, that functional approaches provide promising results for short-term or medium-term forecasts of electricity demand. This is due to their capability of integrating the underlying functional nature of the data when modeling. It is also emphasized that such an approach allows a researcher to consider additional information such as smoothness of the functional parameters, that is not available when using standard time-series techniques. However, the researchers have not addressed the issue when forecasting a series with unusual or outlying observations but have stated that such situations can affect the model accuracy (Antoch *et al.*, 2010). Considering demand data as functions, Shan (2013) has reduced the dimensionality of the curves using functional principal component analysis, where the forecasting was based on the principal component scores. It is to be emphasized that the high accuracies have been obtained eliminating the unusual observations, thus reducing the volatility of the demand series (Shan, 2013). The functional principal component analysis has been used to reduce the dimensionality and a probabilistic forecast has been made based on the derived components of generalized quantile curves. Moreover, it has been shown that this method outperforms many widely used non-functional approaches based on error analysis (Cabrera & Schulz, 2017).

It is evident that the deterministic behaviour of the curves can be better captured using functional data analysis techniques than any other approach. However, the unusual behaviours of the curves are weakly captured using functional data analysis approaches, which may result in low accuracy levels. On the other hand, machine learning approaches have gained much attention due to their ability to capture non-linear and volatile behaviours of data even in load forecasting. Thus, when forecasting a time series with a repetitive pattern, a hybrid approach

combining a functional data analysis approach with an artificial intelligent approach seems to be an ideal hybridization to improve the accuracy substantially. However, there are no forecasting hybridization methods that focusses on high accuracy levels, where such data are at two different levels and also utilizing a statistical approach and an artificial intelligence approach.

Apart from choosing the most appropriate technique to forecast, additional features are commonly used for model improvements, but such information may not always be available at the data level of interest. However, if the available data at different levels are integrated and used effectively for forecasting, the challenges faced with scarce data at specific levels can be overcome. More importantly, this will improve accuracy, as more structural information of the series is used when forecasting. Moreover, implementing dynamism is useful to accommodate real world scenarios as opposed to implementing static models.

Considering all the above-mentioned aspects, this research paper proposes a novel algorithm named as Two Level Neuro-Functional Forecaster that can be used to dynamically forecast functional data at two levels, combining a statistical approach and an artificial intelligence approach.

MATERIALS AND METHODS

The proposed hybrid Two-Level Neuro Functional Forecaster algorithm is an efficient algorithm that can be useful to forecast a time series with a repetitive pattern, where data are at two different levels, even when data are sparse. When considering the hybridization in this algorithm, it utilizes a rule-based approach combining a statistical approach and an artificial intelligence approach as illustrated in Figure 1. The approach will be explained under three subsections namely, required data, techniques used, and rule-based hybridization.

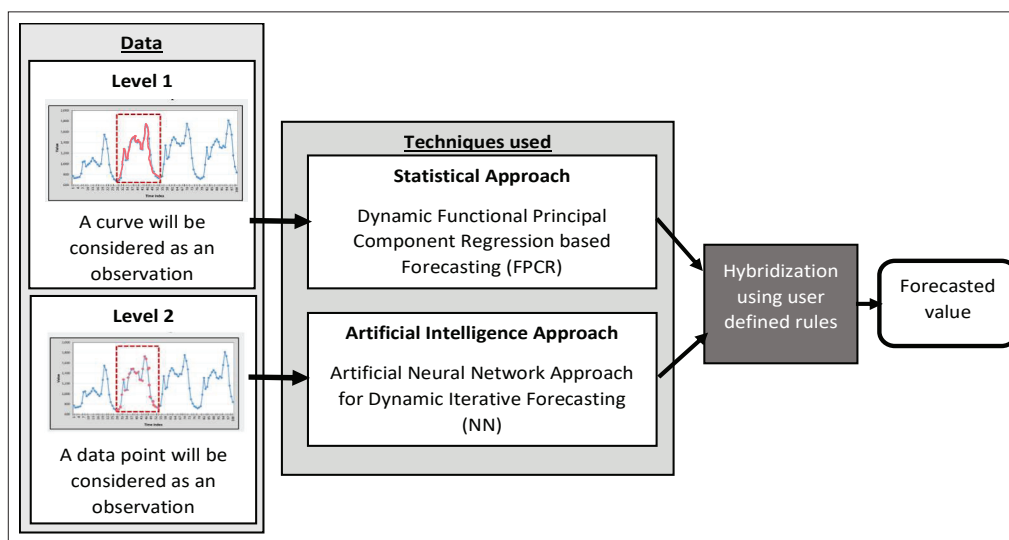


Figure 1: A diagrammatic representation of the proposed hybrid approach

Required data

Certain time series data with a repetitive pattern can be forecasted at two levels depending on the availability of data. Recall that at the higher level, observations are considered as curves, whereas at the lower level, observations are taken at each time point.

When considering the higher level, a statistical approach, developed as Dynamic Functional Principal Component Regression (FPCR), is used to forecast

the series considering the repetitive pattern (Deshani *et al.*, 2017). At the lower level, an artificial intelligence approach, developed as Artificial Neural Network Approach for Dynamic Iterative Forecasting (NN) (Deshani *et al.*, 2019), is utilized to forecast the series.

Techniques used

The two main approaches FPCR and NN used in this Two-Level Neuro Functional Forecaster algorithm are described below.

Dynamic functional principal component regression (FPCR) approach

The FPCR algorithm considers a curve, related to a repetitive pattern, as a single observation where the curves are presented as functional objects. This algorithm consists of three main steps.

- Step 1:** Reducing the dimensionality of the functional objects through functional principal component analysis.
- Step 2:** Forecasting the principal component scores through fitting time series models. A carefully chosen moving window is then used to dynamically forecast the values. That is, more recent data will be incorporated to fit the models while observations prior to the moving window will be disregarded.
- Step 3:** Calculating the future load using the forecasted principal component scores.

Artificial intelligence approach named as artificial neural network approach for dynamic iterative forecasting (NN)

The NN forecasting method, applied to lower-level data, considers many associative variables available at each time point. It utilizes an error backpropagated neural network for an iterative forecast. This algorithm is developed, so that the recently forecasted values are incorporated to the immediate next forecast. Moreover, a moving window is selected to make the forecasting process dynamic. This algorithm consists of four main steps

- Step 1:** Identifying the lagged terms and the features that are influential to the respective time series.
- Step 2:** Identifying the best network architecture for the model and use such multiple neural networks for the forecasting.
- Step 3:** Selecting the best moving window.
- Step 4:** Iteratively forecasting the series.

Rule based hybridization

In this proposed algorithm, the set of rules to be used for the hybridization are formulated by considering the most recent time frame and various characteristics of the errors in each of two approaches. Here, the error bands (first quartile of the error series, third quartile of the error series) of each time point are explored for the two distinct approaches, and the method having a narrow error band centred around zero or the average value of the two forecasts from the two methods is chosen for the final forecast, as appropriate.

Thus, the proposed two-level neuro functional forecaster algorithm can be applied for forecasting a time series that has a structure of a repetitive pattern and where the data are available at two levels.

RESULTS & DISCUSSION

To illustrate this forecasting algorithm, hourly total electricity demand for the period of 1st January 2008 to 31st December 2012 from Sri Lanka was considered as the study was initiated in 2013. The structure of the dataset was explored and is given in Figure 2, where the data are at two levels, namely daily and hourly.

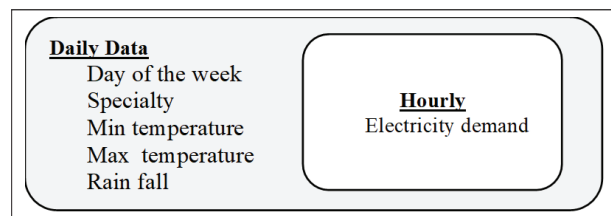


Figure 2: The structure of the dataset at different levels

Furthermore, it consisted of *day of the week* (Sunday, Monday, ..., Saturday), *specialty* (None, Public Holiday, Bank Holiday, Poya day, Saturday after a holiday, 1 day after New Year, 2 days after New Year), *minimum daily temperature*, *maximum daily temperature* and *daily rainfall* at the daily level. The only available hourly measurement was the hourly electricity demand.

Exploring the data

Figure 3 displays the load curves over the study period showing a clear repetitive pattern over the years with a gradual increase of demand, with a very few exceptions. The demand variations are shown through colours of the rainbow where the lowest demand being violet and the highest being red. Recall that a day is considered as the higher level, and an hour is the lower level.

Figure 4 illustrates load curves for a selected time period, separately drawn for the months of January 2008 and January 2012 to elicit the demand variations across days of the week. It can be clearly observed that the demand curve in general has shifted up from 2008 to 2012 but maintaining a similar shape between them. When comparing load curves across the days of the week, demands on Saturdays and Sundays were comparatively lower than other working days of the week, indicating

the importance of using the day effect. Thus, it was decided to perform a comprehensive cluster analysis and to incorporate the results as inputs to the forecasting

models to enhance the accuracy and the efficiency by incorporating them as additional features (Deshani *et al.*, 2014a; 2014b; 2022).

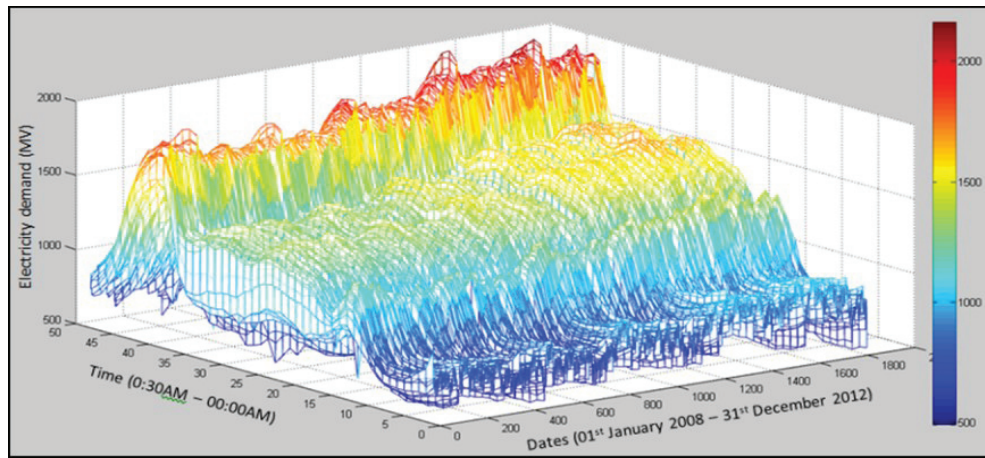


Figure 3: Fluctuations of electricity load curves over the years

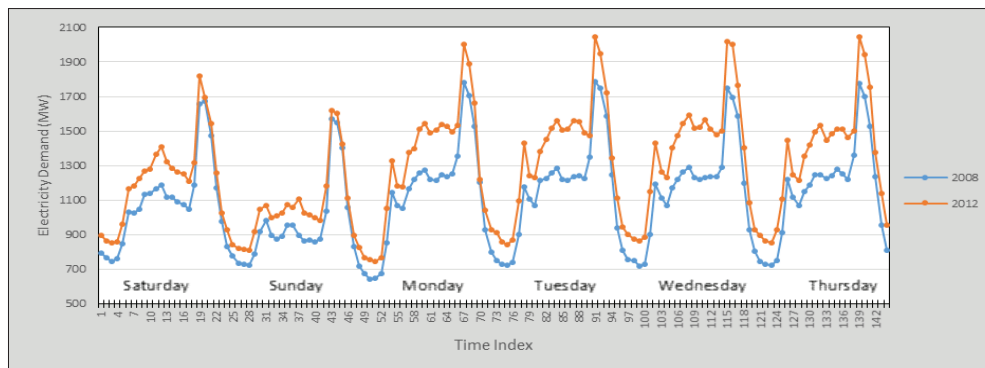


Figure 4: Shape of few selected daily load curves in January 2008 and 2012

Hourly temperatures and rainfall measurements were not available in this data, and thus had to be estimated to be incorporated in this algorithm. The two estimations, respectively, were based on using the MinMax Cos-LEA estimation (Deshani *et al.*, 2016) algorithm and the rainfall adjusted temperature estimation (RATE) algorithm (Deshani *et al.*, 2016). Both algorithms utilize the available daily maximum and minimum temperatures and rainfall data to estimate the hourly temperature measurements.

As such, identifying the key variables separately at both data levels, and preparing data to suit the said data

structure, are an integral part of the process, before applying the two-level neuro functional forecaster algorithm.

Application of the algorithm

The two-level neuro functional forecaster algorithm, as illustrated in Figure 5, accommodates data at two levels using 2 different approaches. First, at the higher level, considering daily load curves as observations, the FPCR approach was used to forecast the demand series using the daily measurements. The data used for this approach were the daily load curves, day of the week, speciality

of the day, daily maximum temperature, daily minimum temperature, curve clustering results and the month, based on this data set. The selected principal component scores were forecasted using the ARIMA model with regressors that best suited this data (Deshani et al., 2017).

Then the NN approach was used to forecast the series considering hourly demand values as separate observations. Selected lagged demand data, demand clustering results and hourly temperatures were used as inputs to this approach. One day ahead hourly demand values were forecasted in an iterative manner and the

findings have been published by the same authors (Deshani et al., 2019).

Consequently, as stated in section ‘Rule based hybridization’, the process of combining the results of the two approaches (FPCR approach and the NN approach) to implement the hybrid forecast was carried out based on carefully chosen rules. These rules were established using the forecasting patterns of the year 2011, the latest available data, through observing the mean absolute percentage error (MAPE) of the dynamic forecasts of the two approaches.

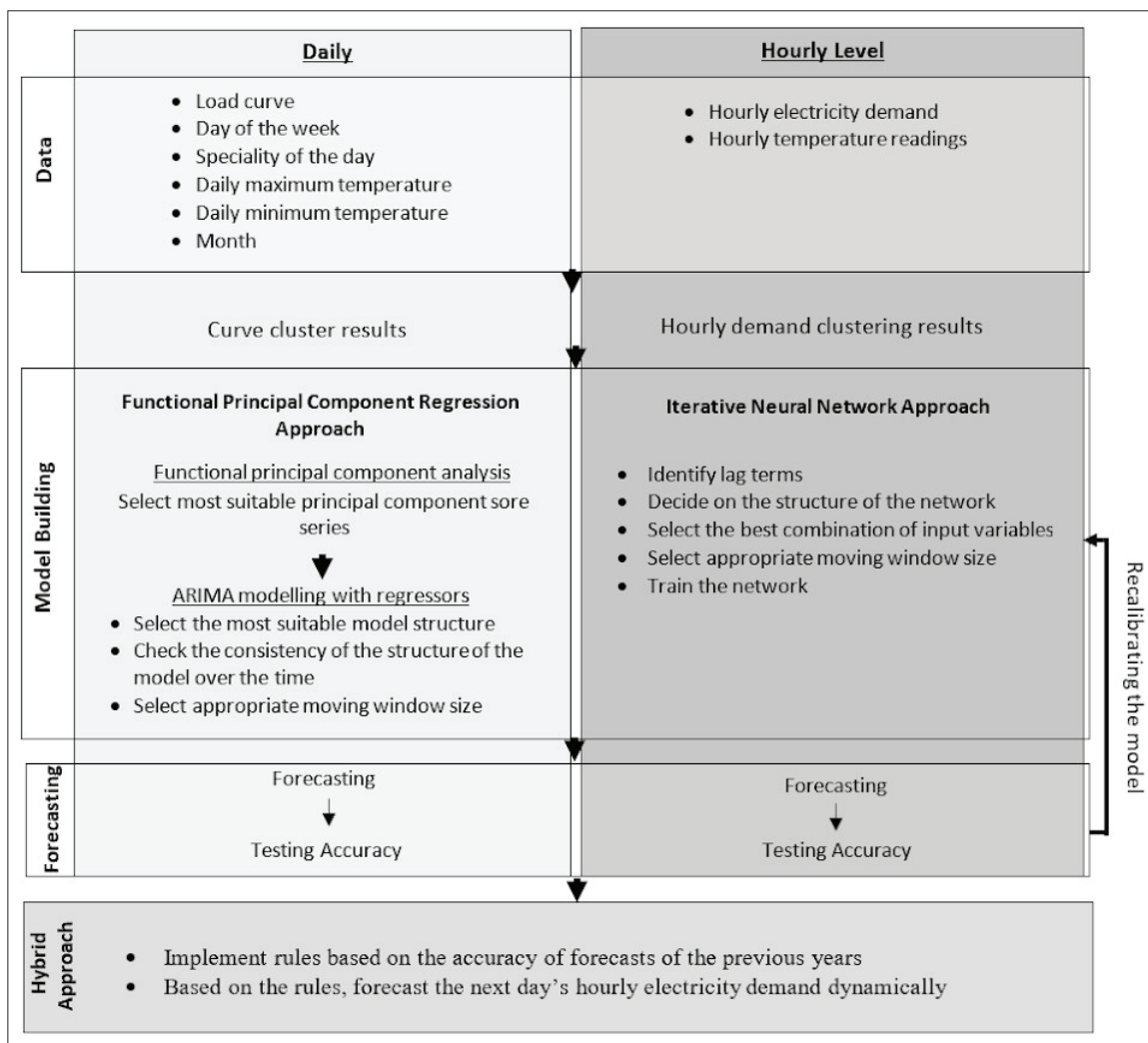


Figure 5: Flow chart of the proposed decision-making algorithm

The outcome of the application of the two approaches was such that, for some day types, the forecast of one approach outperformed the forecast of the other approach. It should be highlighted that the day of the week and the specialty had a strong influence on the demand. Moreover, the time of the day also played a key role when selecting the most suitable approach for the forecast. Thus, it was decided to use a rule-based approach to do the hybridization, focusing on the time of the day, the day of the week and the specialty of the day. The hybridization took place differently for various time periods, choosing the better performing approach or their averages accordingly. Figure 6 clearly shows how the error bands (the first and third quartiles of the hourly

errors) of the two approaches vary with respect to the time of the day. It can be clearly seen that the error bands obtained from one approach become very narrow and centred around zero during some time intervals different from the other approach. Thus, when combining the two approaches, for a particular time period, it was decided to select the better performing approach that had a narrow error band centred around zero to do the forecast. If such a prominent difference cannot be observed, the average forecasted value obtained from the two methods was to be selected. Applying the two approaches in this manner, promising results have been shown, as opposed to using a weighted average of the forecasted values, without any such considerations.

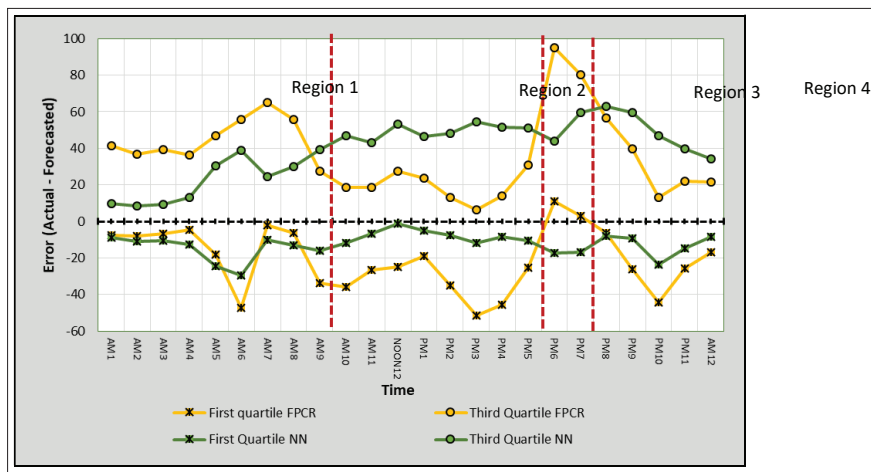


Figure 6: Variations of the first quartile and the third quartile in each hour during 2011

The implemented rules for forecasting were formulated for the preceding entire year paying attention to impacting features, which in this application were the time of the day, the day of the week and the specialty of the day.

After a comprehensive error analysis, the following rules were established and adopted when forecasting the hourly electricity demand.

- Rule 1:** For each day, forecast electricity demands during 1.00 a.m – 9.00 a.m using the NN approach
- Rule 2:** If the specialty of a day is a PBM holiday, Poya day, Saturday after a holiday, 1 day after New Year, or 2 days after New Year, use the NN approach to forecast all 24 hours.
- Rule 3:** If a day is having no specialty, forecast the electricity demands during 10.00 a.m – 5.00 p.m using the average of the forecasts using the FPCR approach and the NN approach.
- Rule 4:** Forecast the electricity demands during 6.00 p.m – 7.00 p.m using the NN approach.

Based on the above-mentioned rules, the final forecasts were made.

The accuracy of the dynamic forecast using two level neuro-functional forecaster

The hourly electricity demand in Sri Lanka for the year 2012 was forecasted using the two level neuro-functional forecaster algorithm considering the rules implemented to do the hybridization. Due to the dynamism in this forecasting algorithm the errors for an entire year were investigated. It was found that the MAPE values were not consistent over the year but gave an overall average of

3.324% for the entire year (Average RMSE for the entire year 55.05). As with much published work in relation to such forecasts, with this data too, the highest errors were reported on holidays where the consumption patterns were highly unpredictable. According to Figure 7, it can be clearly observed that the MAPE for the New Year season was comparatively higher than that of other days whereas the maximum error was observed for the day after the New Year. Figure 8 displays some selected forecasted daily load curves with different MAPEs.

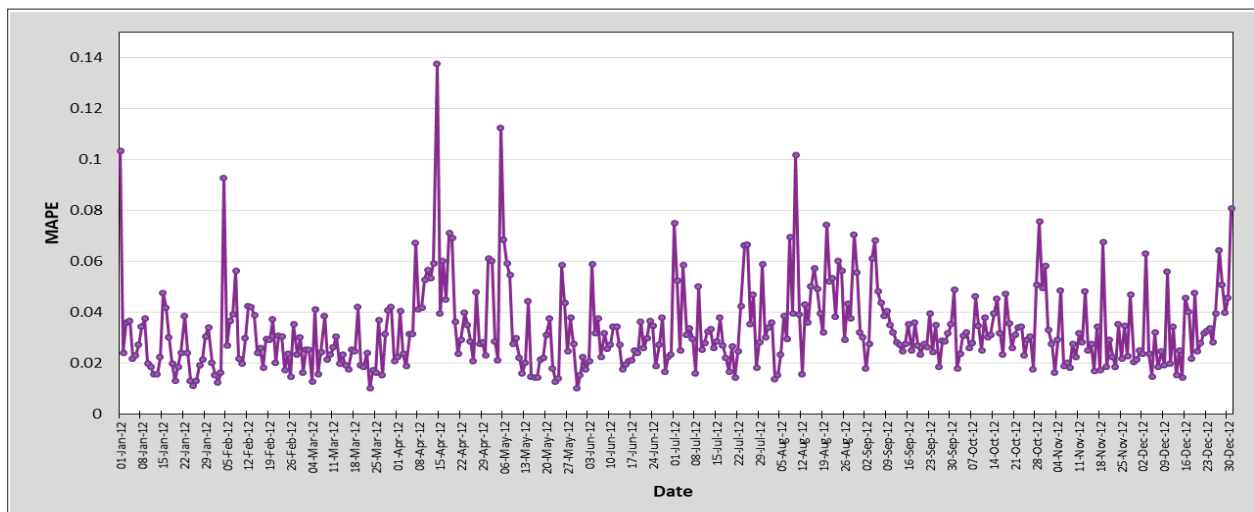


Figure 7: MAPE of the final dynamic forecasting model

A comparison of dynamic forecasts for the year 2012 using the FPCR approach, NN approach and the proposed two level neuro-functional load forecaster approach is depicted in Figure 9. It can be clearly seen that using the proposed hybrid approach, the best forecasts can be made, as opposed to when the approaches are used separately as two single approaches. Wilcoxon sign-rank test for paired samples were used to statistically assess whether the median MAPE of the two individual approaches are significantly higher than the median MAPE of the hybrid forecasts. Wilcoxon signed-rank test showed that the two approaches elicited a statistically significant increase in the MAPEs compared to the MAPE of the hybrid forecasting (p -value = $1.108e-12$ when comparing with the NN approach and p -value $< 2.2e-16$ when comparing with the FPCR approach). Thus, the two level neuro-functional forecaster approach yields superior results compared to the individual forecasts from the two approaches.

Increasing the accuracy levels of the forecasting through hybrid approaches has gained much attention in the recent past. This research highlights the importance of exploring the structural aspects and other features of the time series and incorporating them when selecting the most appropriate forecasting models for hybridization. One such important aspect is to view the structure of the time series with a repetitive pattern, as data from two different levels, that will allow the capture of unique features from each level, effectively. There can be other types of time series where no such structural patterns can be easily identified. Then it would not be possible to enhance the accuracy of the forecasts, by incorporating level specific information to the models. Through the electricity demand data application presented in this paper, the structural behaviour can be easily captured at the daily level, while the unusual sudden behaviours can be easily accommodated at the hourly level. With the use of the features at two levels, the thorny issue of

data scarcity can also be addressed in an effective way as one level will facilitate the model with the required

data, though the other level does not comprise the data as required.

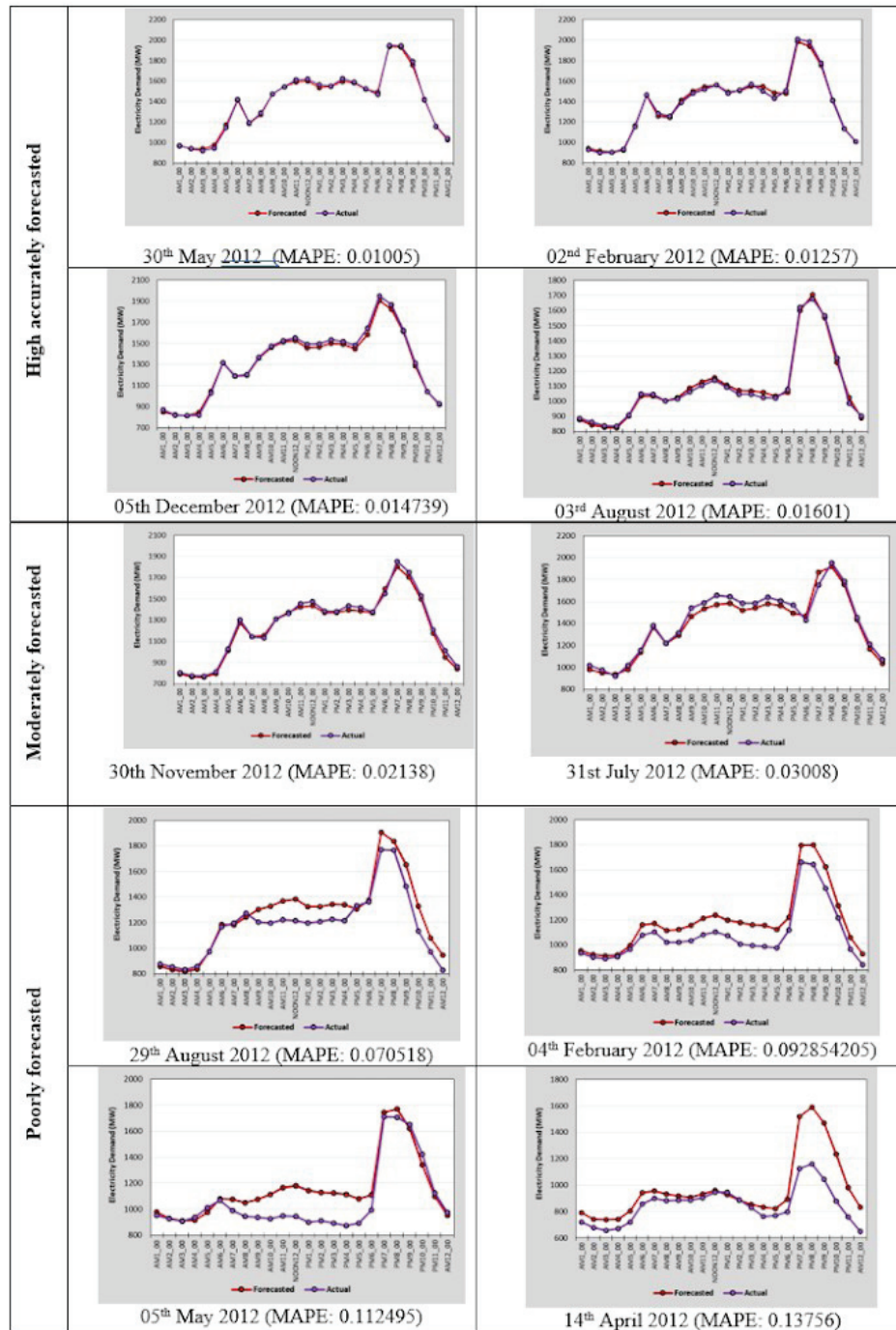


Figure 8: Some well, moderately and poorly forecasted days using the hybrid approach

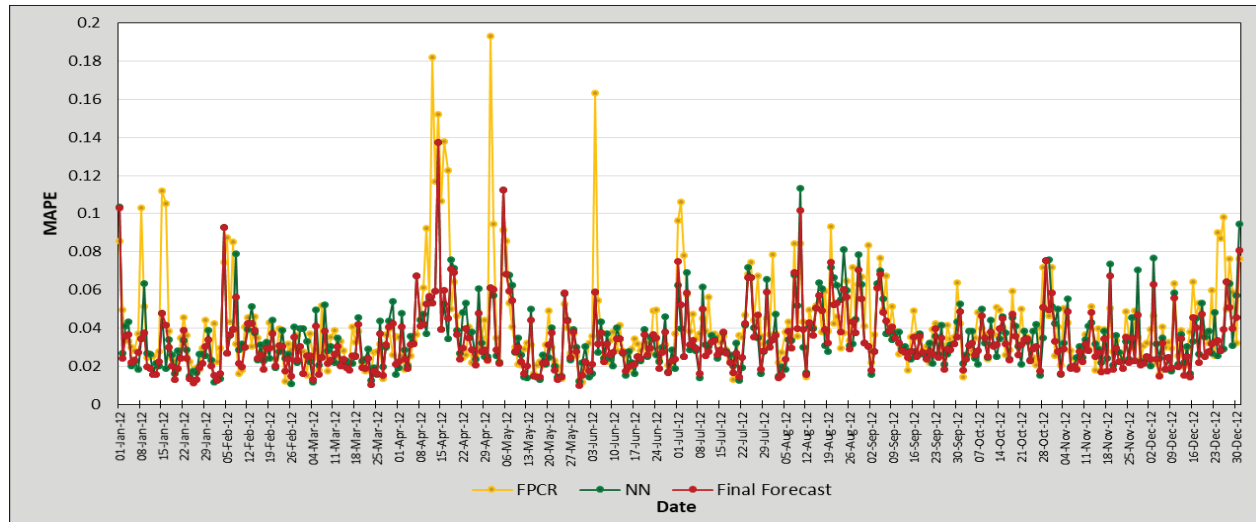


Figure 9: MAPE of the 2012 dynamic forecasts from the three methods

CONCLUSION

When implementing hybrid models, it should be highlighted that approaches from two different fields can be fruitfully utilized to overcome the weaknesses of one approach compared to the other. Thus, considering all the above-mentioned aspects and also with proven promising results, the proposed two level neuro-functional forecaster algorithm can be recommended as an ideal forecasting algorithm for a time series with a repetitive pattern where the data are available at two levels. For the application presented in this paper, the dynamic forecast yielded an average MAPE of 3.32% for an entire year while the FPCR approach and the NN approach resulted in a MAPE of 4.08% and 3.96%, respectively, for the entire year when used separately. Thus, it can be concluded that the proposed dynamic hybrid algorithm can be used to gain high accuracies for time series data with repetitive patterns rather than using the FPCR approach or the NN approach with the use of the concept of data at two levels. This study sets the platform to explore various rule-based hybridization techniques to forecast complex time series data.

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